1. **[4 marks – 2, 2]**

a) If anm is the element situated in matrix A, write down matrix A given that it is a 3x2 matrix with anm= n2 - 2m

|  |  |
| --- | --- |
| **Solution** | |
| Correct dimensions (3x2) | 1 mark |
| Correct values | 1 mark |

b) Given D = and E = , calculate the value of **2D-E.**

|  |  |
| --- | --- |
| **Solution** | |
| Correct calculates 2D | 1 mark |
| Correctly states solution | 1 mark |

1. **[5 marks – 2, 3]**

Let **A** =  and  **B** = 

**a)** Find the value(s) of if **A** is non-singular matrix.

|  |  |
| --- | --- |
| **Solution** | |
| Correctly states the determinant expression not equal to 0 | 1 mark |
| Correctly states solution | 1 mark |

**b)** Find the value(s) of if **AB** = **A** + **B**

|  |  |
| --- | --- |
| **Solution**  (Can be stated as mixed ) | |
| Calculate A+B | 1 mark |
| Calculates AB | 1 mark |
| Correctly calculates | 1 mark |

1. **[4 marks – 1, 1, 2 ]**

Let. **A =** , **B** = , **C** = , **D** = 

**a)** Find |**D**|, the determinant of D.

|  |  |
| --- | --- |
| **Solution** | |
| Correct calculates determinant | 1 mark |

**b)** Find the value of and if **D**-1 = 

|  |  |
| --- | --- |
| **Solution** | |
| Correct calculates | 1 mark |

**c)** It is possible to form the product of all four matrices. Determine in what order this can be done and state the dimensions of the resulting product. (*Do not calculate the matrix*)

|  |  |
| --- | --- |
| **Solution**  A (3x2) B(1x2) C(2x1) D(2x2)  YES- ACBD gives a 3 x 2 matrix OR ADCB gives a 3 x 2 matrix | |
| Correct states Yes and the at least one example example of a correct product order | 1 mark |
| Correctly states the dimensions of above product | 1 mark |

1. **[6 marks – 1, 1, 2, 2]**

Let **A** = .

1. Find **A**-1

|  |  |
| --- | --- |
| **Solution** | |
| Correct calculates A-1 | 1 mark |

1. Find the product of **A**-1

|  |  |
| --- | --- |
| **Solution** | |
| Correct calculates the solution | 1 mark |

1. Consider the system of equations:

* 1. Rewrite the given system of equations in the form **BX** = **C** where **X** is a column matrix and **B** and **C** are appropriate matrices.

|  |  |
| --- | --- |
| **Solution** | |
| Correct matrices B, C and X | 1 mark |
| Correctly writes the system of equation | 1 mark |

* 1. Use a matrix method to solve for and .

|  |  |
| --- | --- |
| **Solution** | |
| Demonstrates the use of X = A-1B | 1 mark |
| Correctly states solution | 1 mark |

1. **[6 marks – 3, 3]**
2. Prove

|  |  |
| --- | --- |
| **Solution**  = RHS | |
| Chooses common denominator | 1 mark |
| Calculates difference of squares to find cos2 | 1 mark |
| Simplifies the numerator | 1 mark |

1. Prove the identity

|  |  |
| --- | --- |
| **Solution**  = RHS | |
| States/identifies appropriate reciprocal identities | 1 mark |
| States/identifies appropriate double angle identities | 1 mark |
| Correctly proves identity |  |



**Marks for Section 2:** *31 marks*

1. **[9 marks - 2, 2, 2, 3]**

Let **A** = and **B** = .

1. Given that , determine the value of *k*.

|  |  |
| --- | --- |
| **Solution** | |
| Calculate A-1 correctly | 1 mark |
| Correctly states solution | 1 mark |

1. The equations and can be expressed as a matrix equation in the form **AX=C**

**i)** State the matrices **X** and **C**.

|  |  |
| --- | --- |
| **Solution** | |
| Correctly rearrange equations to reflect matrix A | 1 mark |
| Correctly states X and C | 1 mark |

**ii)** Write down a matrix equation to determine **X**in terms of **B** and **C**.

|  |  |
| --- | --- |
| **Solution**  **X=A-1C**  **X = BC** | |
| States or identifies the inverse of A | 1 mark |
| Correctly states solution in terms **B** and **C** | 1 mark |

1. Determine the matrix **D**, if **(B + D)B = 3A**

|  |  |
| --- | --- |
| **Solution**  **B+D = 3AB-1**  **D = 3AB-1 – B**  **=** | |
| Identifies the B-1 | 1 mark |
| Rearranges to subtract B | 1 mark |
| Calculates the correct matrix D | 1 mark |

1. **[6 marks – 1, 2, 1, 2]**

M & L Fund Managers buy and sell shares in companies on the stock market. They have three traders working for them, Jim, Katey and Anton.

The table below shows the number of shares in some stocks held in each of the three traders accounts at the **end** of the week

|  |  |  |  |
| --- | --- | --- | --- |
| Stock | Jim | Katey | Anton |
| AGO | 4500 | 1900 | 5400 |
| GXY | 4800 | 4500 | 3000 |
| MBN | 4150 | 8050 | 5850 |

1. Represent the table above by a 3 x 3 matrix **A**

|  |  |
| --- | --- |
| **Solution** | |
| Correctly states matrix A | 1 mark |

Matrix **X** below represents the change in the number of each type of share over the week for the four traders.

1. Use Matrices **A** and **X** to find matrix **S**, the shares held in each stock by each trader at the **start** of the week.

|  |  |
| --- | --- |
| **Solution**  **S = A-X**  = | |
| Correctly identifies the subtraction model | 1 mark |
| Calculates matrix S correctly | 1 mark |

The share price at the end of the week for each of these shares is below.

**AGO** - $1.655, **GXY** - $0.625, **MBN** - $2.560

1. Write these as Matrix **P**

|  |  |
| --- | --- |
| **Solution** | |
| Correctly states the matrix P | 1 mark |

1. Use the previous matrices to calculate the total value of each trader’s holdings at the **end** of the week.

|  |  |
| --- | --- |
| **Solution**  Jim $21,071.50  Katey $26,565  Anton $25,788 | |
| Use the correct calculation | 1 mark |
| Clearly states the total value of each trader | 1 mark |

1. **[3 marks]**

Given Matrix **M** is 2 x 2 and **M**2 – **M** + 2**I** = 0, show that **M**4 + 3**M** + 4**I** = 0

|  |  |
| --- | --- |
| **Solution**  Given M2-M+2I = 0, then M2=M-2I  M4+3M2+4I = (M-2I)2+3M2+4I  = M2-2M-2M+4I+3M2+4I *(note - MI and IM = M)*  *= 4M2-4M+8I*  = 4(M2-M+2I)  = 4 (0) = 0 | |
| Rearranges given information | 1 mark |
| Substitutes and expands correctly | 1 mark |
| Factorise and finishes demonstration | 1 mark |

1. **[5 marks ]**

A nut distributor wants to know the nutritional content of various mixtures of almonds, cashews and pecans. Her supplier has provided the following information:

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Almonds** | **Cashews** | **Pecans** |
| **Protein** (g/cup) | 26.2 | 21 | 10.1 |
| **Carbs** (g/cup) | 40.2 | 44.8 | 14.3 |
| **Fat** (g/cup) | 71.9 | 63.5 | 82.8 |

Her first mixture, a protein blend consists of 6 cups of almonds, 3 cups of cashews, and 1 cup of pecans. Her second mixture, a low fat mix, consists of 3 cups of almonds, 6 cups of cashews, and 1 cup of pecans. Her third mixture a low carb mix consists of 2 cups of almonds, 1 cup of cashews, and 7 cups of pecans.

Use appropriate matrix calculations to determine the amount of protein, carbs and fats in a **1 cup** serving of each of the mixtures.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Solution**  Let  A represent the nutritional content of the various nuts  B represent the cups of various nuts for 10 cups of each mixture    Nutritional content of g/1 cup of each mixture = AB =   |  |  |  |  | | --- | --- | --- | --- | |  | Protein Mix | Low Fat Mix | Low Carb Mix | | Protein g/cup | 23.03 | 21.47 | 14.41 | | Carb g/cup | 38.99 | 40.37 | 22.53 | | Fat g/cup | 70.47 | 67.95 | 78.69 | | |
| States the matrix for given table | 1 mark |
| Creates and appropriate matrix for mix ratios | 1 mark |
| Uses an appropriate matrix multiplication technique | 1 mark |
| Uses scalar multiplication to convert to 1 cup measure | 1 mark |
| Clearly states the solution in table or appropriate format |  |

1. **[4 marks – 2, 2]**

The following table gives the number of people (in thousands) who visited Australia and South Africa in 2016.

|  |  |  |
| --- | --- | --- |
|  | **Australia** | **South Africa** |
| *From* **North America** | 440 | 190 |
| **Europe** | 950 | 950 |
| **Asia** | 1790 | 200 |

You estimate that from 2016 to 2021 tourism from North America to each of Australia and South Africa is predicted to increase by 15%, tourism from Europe by 25%, and tourism from Asia will decrease by 10%. Take **A** to be the 3 x 2 matrix whose entries are the 2016 tourism figure in the above table.

1. Write the estimated changes as matrix **B,** and show the calculation for the total predicted tourists from North America, Europe and Asia to Australia and South Africa.

|  |  |
| --- | --- |
| **Solution**  BA = | |
| Clearly identifies an appropriate matrix B | 1 mark |
| Clearly shows the calculation used to predict the tourists | 1 mark |

1. How many tourists are predicted to come to travel to South Africa from Asia in 2021?

|  |  |
| --- | --- |
| **Solution**  BA = From matrix 2021 prediction is 180, 000 | |
| Identifies correct value from matrix | 1 mark |
| Clearly states the correct prediction in thousands | 1 mark |

1. **[3 marks]**

Prove the identity

|  |  |
| --- | --- |
| **Solution**  =RHS | |
| Expand angle sum identity | 1 mark |
| Simplify fraction | 1 mark |
| Clearly prove identity | 1 mark |